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Robust Spectral Clustering for Noisy Data

Modeling Sparse Corruptions Improves Latent Embeddings

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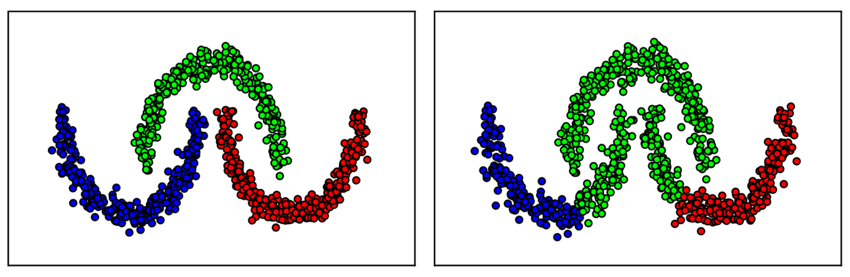
MOTIVATION

- Spectral clustering (SC) **widely** used, but highly **sensitive** to noisy data
- Noise distorts the embedding space and obfuscates the clustering structure

Noise 0.07 | Noise 0.0875



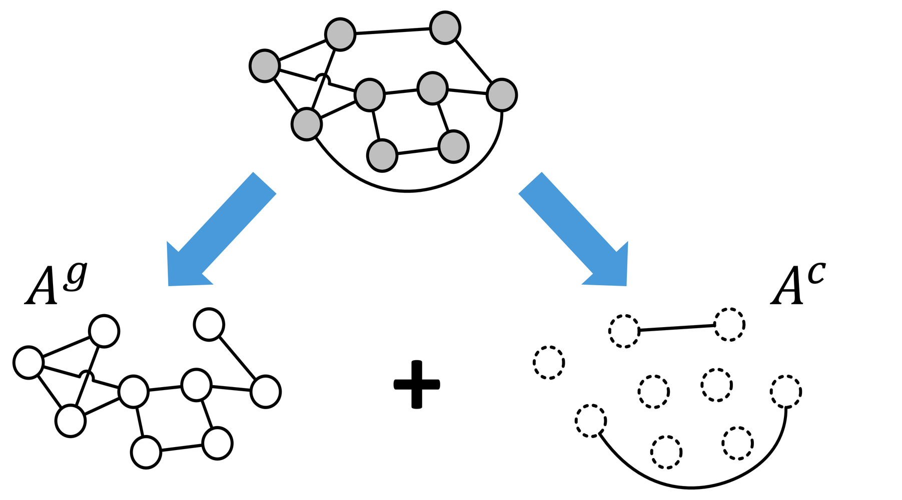
SC ✓ RSC ✓ | SC ✗ RSC ✓

- We propose a robust version: **RSC**

PROBLEM FORMULATION

❑ Core idea: **Latent Decomposition**

$$A = A^g + A^c$$



❑ Jointly learn decomposition and embedding

- Decomposition steered by the underlying clustering

$$A^*, H^* = \underset{H\in\mathbb{R}^{n\times d},\ A^g\in(\mathbb{R}_{\geq 0})^{n\times n}}{\operatorname{argmin}} \ Tr(H^T \cdot L(A^g)\cdot H) \qquad (1)$$

subject to: $H^T \cdot D(A^g)\cdot H = I$ and

$A = A^g + A^c, \|A^c\|_0 \leq 2\theta, \|a_i^c\|_0 \leq \omega_i$

- Robust formulation for all SC versions
- Result → improved embedding

ALGORITHMIC SOLUTION

Alternating Optimization

❑ Update H, Given A^g/A^c → ***Easy***

- Trace minimization problem
- Solution for H are the k first generalized eigenvectors of $L(A^g)$

❑ Update A^g/A^c, Given H → ***(NP) Hard***

- Express eigenvalues of A^g_{new} in closed form
- A^g_{new} that minimizes (1) equivalent to maximizing:

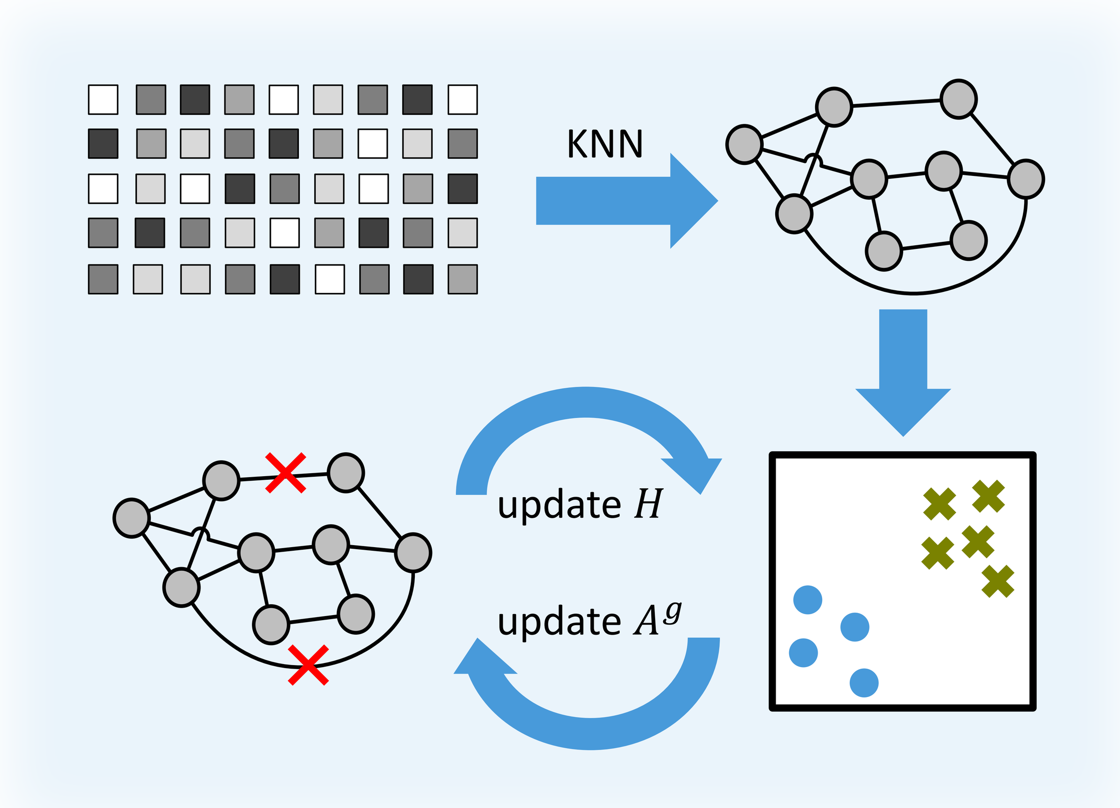
} Step 1: Eigenvalue Perturbation

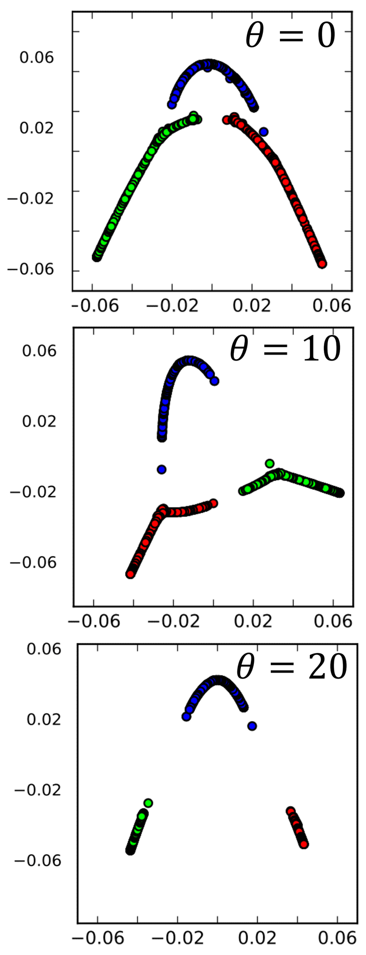
$$f([a^c_{uv}]_{u,v\in E}) = \sum_{u,v\in E} a^c_{uv}\left(\underbrace{\|\boldsymbol{h}_u - \boldsymbol{h}_v\|_2^2}_{nodes\ far\ away\ in\ the\ embedding\ space} - \underbrace{\left\|\sqrt{\lambda}\circ \boldsymbol{h}_u\right\|_2 - \left\|\sqrt{\lambda}\circ \boldsymbol{h}_v\right\|_2}_{prefers\ edges\ close\ to\ critical\ region}\right)$$

subject to $\|\cdot\|_0$ constraints

- Observation: Above problem is equivalent to **Multidimensional Knapsack** problem
- Greedy approximation scheme
- Best possible approximation ratio of $1/\sqrt{N+1}$

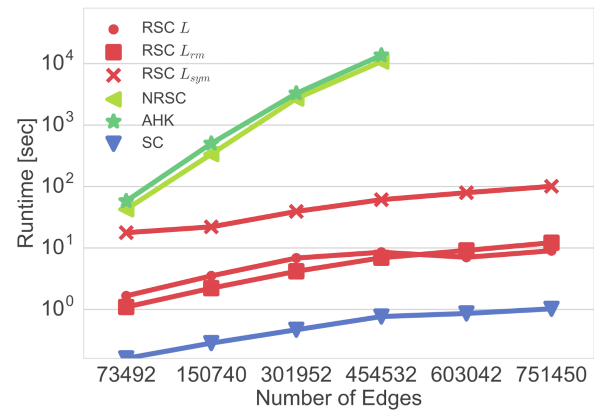
} Step 2: Efficient Solution





❑ Complexity

- Derivations for all 3 established versions of SC (different Laplacians)
- Each version has linear runtime in the number of edges $O(E)$



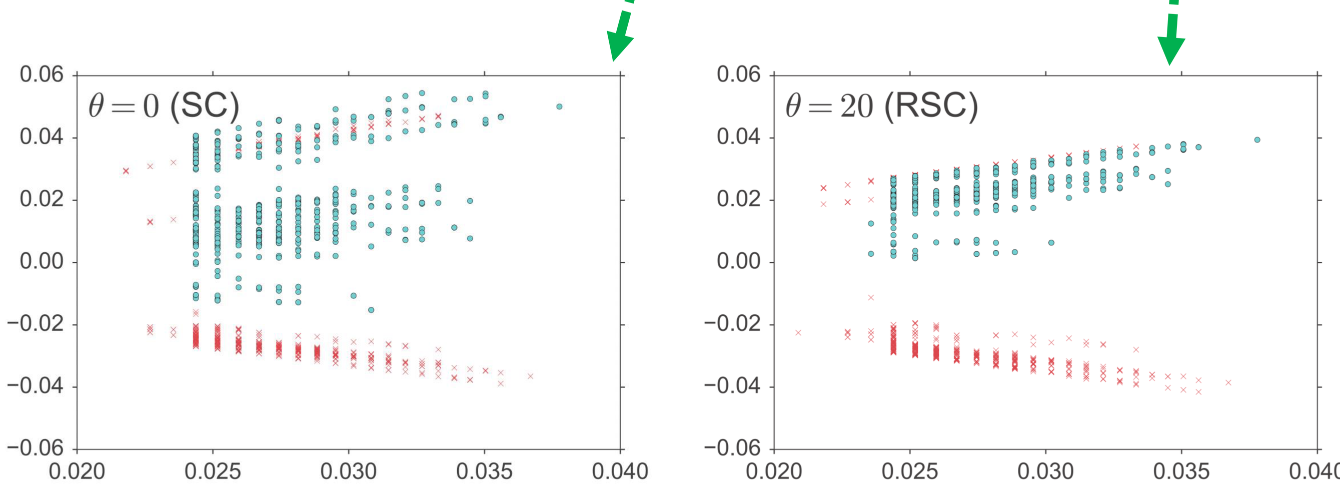
RESULTS

How can we evaluate the quality of the **clustering**?

❑ RSC improves the clustering as measured by the NMI

Dataset	AHK	NRSC	SC		RSC
Moons	0.53	0.99	0.47	+112.8 %	1.00
Banknote	0.53	0.47	0.46	+ 32.6 %	0.61
USPS	0.77	0.83	0.78	+ 8.9 %	0.85
MNIST	0.71	0.76	0.70	+ 11.4 %	0.78
Pendigits	0.94	0.94	0.93	+ 3.2 %	0.96

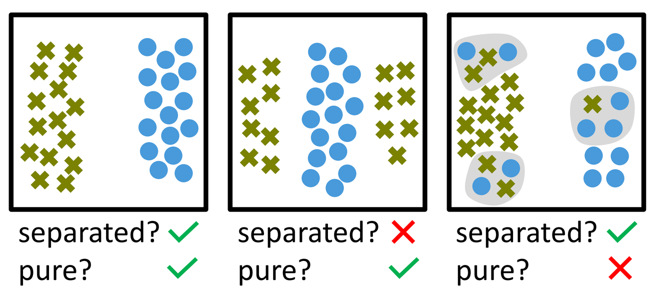
❑ RSC improves discrimination in the embeddding space



How can we evaluate the quality of the **embeddings**?

❑ Global Separation

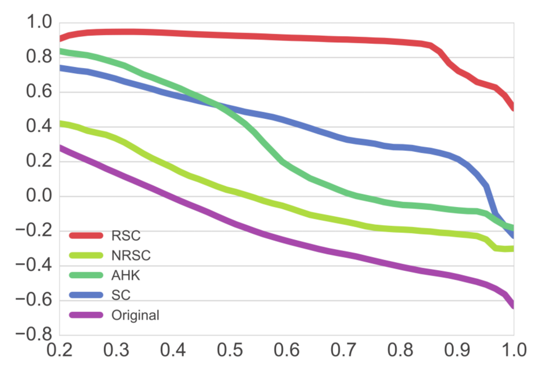
- Degree of separability between clusters
- Robust silhouette coefficient



$$P_{c,c'}(x) = \underset{x\,\%\,smallest}{\text{average}}\left[dist(\boldsymbol{h}_i,\boldsymbol{h}_j)\right]_{i\in C_c, j\in C_{c'}}$$

$$GS_c(x) = \frac{P_{c,c'}(x) - P_{c,c}(x)}{\max\{P_{c,c'}(x), P_{c,c}(x)\}}$$

$$c' = \underset{c\neq c'}{\operatorname{argmin}}\, P_{c,c'}(x)$$



(worst) $-1 \leq GS_c(x) \leq 1$ (best)

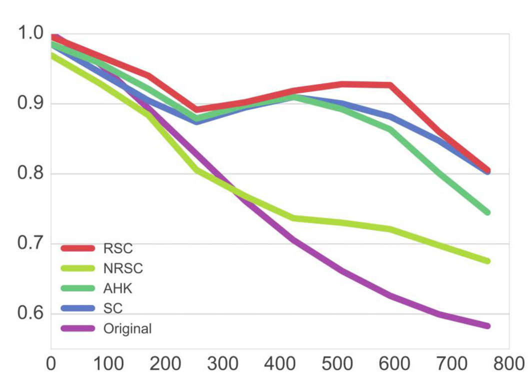
❑ Local Purity

- Homogeneous local neighborhood

$$occ_x(c,i) = \left|\{ j \in NN_x(i) \cup \{i\} \mid c_j = c\}\right|$$

$$pur_x(i) = \frac{1}{x+1}\max_{c\in C} occ_x(c,i)$$

$$\text{PUR}(x) = \frac{1}{N}\sum_{i=1}^{N} pur_x(i)$$



(worst) $\frac{1}{|C|} \leq PUR(x) \leq 1$ (best)